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Adding a brane to the brane-anti-brane action in BSFT

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Abstract: We attempt to generalize the effective action for the D-brane-anti-D-brane system obtained from boundary superstring field theory (BSFT) by adding an extra D-brane to it to obtain a covariantized action for 2 D-branes and 1 anti-D-brane. We discuss the approximations made to obtain the effective action in closed form. Among other properties, this effective action admits solitonic solutions of codimension 2 (vortices) when one of the D-brane is far separated from the brane-anti-brane pair.

Keywords: Superstrings and Heterotic Strings, D-branes, Tachyon Condensation, String Field Theory.

Contents

1. Introduction

The study of non-BPS brane system has revealed a lot of interesting properties of the superstring theory. In particular, the tachyon rolling [1, 2] in the non-BPS-D-brane and the D-brane-anti-D-brane (D$\overline{\mathrm{D}}$) pair allows the study of string dynamics. A particularly powerful approach is the effective actions [3]–[5] derived from boundary superstring field theory (BSFT) [6]–[8]. It is natural to generalize the approach to more complicated systems. In this paper, we attempt to derive the effective action for the 2 D-branes and 1 anti-D-brane (DD$\overline{\mathrm{D}}$) system in BSFT. There are a number of motivations to study this system. The system is more complicated and we like to see how that translates into the formalism and the formulation of the effective action. We also like to use the condensation of the tachyon in this system to produce an action for other systems, for example, that of a Dp-brane plus a D$(p-2)$-brane system, where the D$(p-2)$-brane is a codimension-2 topological defect resulting from the tachyon rolling. On the more phenomenological side, we see that the decay of the D$\overline{\mathrm{D}}$ pair in the DD$\overline{\mathrm{D}}$ system produces both open string modes as well as closed string modes, a feature that is absent in the D$\overline{\mathrm{D}}$ system since there is no open strings modes left after the annihilation of the branes. This property is particularly important in brane inflation, where we like to see most of the energy released from brane collisions to go to reheating the universe, that is, to open string modes instead of closed string modes. This is required by the big bang nucleosynthesis.

Apriori, it is clear that the $\mathrm{DD\overline{D}}$ system involves non-abelian gauge theory and any effective action one can obtain in closed form is a poorer approximation to the actual theory than that for the $\mathrm{D\overline{D}}$ action. Fortunately, there are still many interesting features maintained in a simplified closed form $\mathrm{DD\overline{D}}$ effective action.

The paper is organized as follows. In section 2, we briefly review the BSFT derivation of the $\mathrm{D\overline{D}}$ effective action. In section 3, we present the covariantized $\mathrm{DD\overline{D}}$ effective action. As a check, we show how our new action reduces correctly to the $\mathrm{D\overline{D}}$ effective action when one of the D-brane is moved to ∞. We leave the somewhat lengthy determination of invariants to the appendix. In section 4, we construct the effective action for two Dp-branes and one $\overline{\mathrm{D}}p$-brane by T-dualizing the $\mathrm{D\overline{D}}$ effective action. This allow us to physically justify our approximate effective action. Finally, we study the solitonic solutions of our action in section 5. Section 6 is the conclusion.

2. Review of BSFT and the $\mathrm{D\overline{D}}$ system

We review the $\mathrm{D\overline{D}}$ effective action from BSFT derived in ref [3, 4]. We restrict our attention to D9-branes in type IIB theory. BSFT essentially extends the sigma-model approach to string theory[9], in that (under certain conditions [6, 7]) the disc world-sheet partition function with appropriate boundary insertions gives the classical spacetime action. This framework for the bosonic BSFT was extended to the open superstring in [8] and formally justified in [10, 11]. In the NS sector the spacetime action is

$$S_{\mathrm{spacetime}} = -\int \mathcal{D}X\mathcal{D}\psi\mathcal{D}\tilde{\psi}\; e^{-S_\Sigma - S_{\partial\Sigma}}\,, \tag{2.1}$$

where Σ is the worldsheet disc and $\partial\Sigma$ is its boundary. The worldsheet bulk disc action is the usual one

$$\begin{aligned} S_\Sigma &= \frac{1}{2\pi\alpha'}\int d^2z\; \partial X^\mu \overline{\partial} X_\mu + \frac{1}{4\pi}\int d^2z\left(\psi^\mu\overline{\partial}\psi_\mu + \tilde{\psi}^\mu\partial\tilde{\psi}_\mu\right) \\ &= \frac{1}{2}\sum_{n=1}^{\infty} nX^\mu_{-n}X_{n\,\mu} + i\sum_{r=1/2}^{\infty}\psi^\mu_{-r}\psi_{r\,\mu}\,, \end{aligned}$$

after expanding the fields in the standard modes. To reproduce the Dirac-Born-Infeld (DBI) action for a single brane, the appropriate boundary insertion is the boundary pullback of the U(1) gauge superfield to which the open string ends couple; for the N brane M anti-brane system, the string ends couple to the superconnection [12, 13], hence the boundary insertion should be

$$e^{-S_{\partial\Sigma}} = \mathrm{Tr}\hat{P}\exp\left[\int d\tau d\theta\mathcal{M}(\mathbf{X})\right], \qquad \mathcal{M}(\mathbf{X}) = \begin{pmatrix} iA^1_\mu(\mathbf{X})D\mathbf{X}^\mu & \sqrt{\alpha'}T^\dagger(\mathbf{X}) \\ \sqrt{\alpha'}T(\mathbf{X}) & iA^2_\mu(\mathbf{X})D\mathbf{X}^\mu \end{pmatrix}, \tag{2.2}$$

where the insertion must be supersymmetrically path ordered to preserve supersymmetry and gauge invariance. $A^{1,2}$ are the U(N) and U(M) connections, and T is the tachyon matrix transforming in the $(N,\overline{M})$ representation of $\mathrm{U}(N)\times\mathrm{U}(M)$. The lowest component

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of $\mathcal{M}$ is proportional to the superconnection. To proceed, it is simplest to perform the path-ordered trace by introducing boundary fermion superfields [14]; we refer the reader to [3] for details. The insertion (2.2) can then be simplified to be

$$\mathrm{Tr} P \exp \left[i\alpha' \int d\tau \begin{pmatrix} F^1_{\mu\nu}\psi^\mu\psi^\nu + iT^\dagger T + \frac{1}{\alpha'}A^1_\mu \dot{X}^\mu & -iD_\mu T^\dagger \psi^\mu \\ -iD_\mu T \psi^\mu & F^2_{\mu\nu}\psi^\mu\psi^\nu + iTT^\dagger + \frac{1}{\alpha'}A^2_\mu \dot{X}^\mu \end{pmatrix} \right], \quad (2.3)$$

where the tachyon covariant derivatives are

$$D_\mu T = \partial_\mu T + iA^1_\mu T - iTA^2_\mu = \partial_\mu T + iA^-_\mu T\,. \quad (2.4)$$

This expression reproduces the expected results when the tachyon and its derivatives vanish: the only open string excitations will be the gauge fields on the branes and the anti-branes, for each of which the action is the standard DBI action. For a single brane anti-brane pair, $N = M = 1$, demanding that the gauge field to which the tachyon couples vanishes, $A^- = 0$, the path-ordered trace can be performed using worldsheet boundary fermions. Writing $A^+ = A^1 + A^2$, we have

$$\begin{aligned} S_{\partial\Sigma} &= -\int d\tau \left[\alpha' T\overline{T} + \alpha'^2 (\psi^\mu \partial_\mu T) \frac{1}{\partial_\tau} (\psi^\nu \partial_\nu \overline{T}) + \frac{i}{2}\left(\dot{X}^\mu A^+_\mu + \frac{1}{2}\alpha' F^+_{\mu\nu}\psi^\mu\psi^\nu \right) \right], \\ \frac{1}{\partial_\tau} f(\tau) &= \int d\tau' f(\tau') \mathrm{sgn}(\tau - \tau')\,, \end{aligned} \quad (2.5)$$

where $\mathrm{sgn}(x) = 1$ for $x > 0$ and $= -1$ for $x < 0$. For linear tachyon profiles, spacetime rotations allow us to write $T = u_1 X^1 + iu_2 X^2$, and (2.1) can be calculated, since the functional integrals are all gaussian. The result when $A^+ = 0$ is derived in [3, 4]:

$$S_{\mathrm{D}\overline{\mathrm{D}}} = -2\tau_9 \int d^{10}X_0 \; \exp\left[-2\pi\alpha' [(u_1 X^1_0)^2 + (u_2 X^2_0)^2] \right] \mathcal{F}(4\pi\alpha'^2 u_1^2)\, \mathcal{F}(4\pi\alpha'^2 u_2^2)\,, \quad (2.6)$$

where the function $\mathcal{F}(x)$ is given by [8]

$$\mathcal{F}(x) = \frac{4^x x \Gamma(x)^2}{2\Gamma(2x)} = \frac{\sqrt{\pi}\Gamma(1+x)}{\Gamma(\frac{1}{2}+x)}. \quad (2.7)$$

$$\mathcal{F}(x) = \begin{cases} 1 + (2\ln 2)x + \mathcal{O}(x^2), & 0 < x \ll 1, \\ \sqrt{\pi x} & x \gg 1\,. \end{cases} \quad (2.8)$$

The RR coupling of the D-branes can also be written down. The bulk contribution to the partition sum can be written as the wave-functional [3, 4]

$$\begin{aligned} \Psi^{RR}_{\mathrm{bulk}} &\propto \exp\left[-\frac{1}{2}\sum_{n=1}^{\infty} n X^\mu_{-n} X_{n\,\mu} - i \sum_{n=1}^{\infty} \psi^\mu_{-n}\psi_{n\,\mu} \right] C\,, \\ C &= \sum_{\mathrm{odd}\ p} \frac{(-i)^{\frac{9-p}{2}}}{(p+1)!} C_{\mu_0\cdots\mu_p} \psi_0^{\mu_0} \cdots \psi_0^{\mu_p}\,. \end{aligned}$$

The ψ_0^μ are the zero modes of the Ramond sector fermions, and $C_{\mu_0\cdots\mu_p}$ are the even RR forms of IIB string theory. The normalization of Ψ can be set later by demanding that the correct brane charge is reproduced. The Chern-Simon-like action is then defined by

$$S_{\rm RR} = \int \mathcal{D}X\mathcal{D}\psi\ \Psi^{RR}_{\rm bulk}{\rm Tr}^*\ Pe^{-S_{\partial\Sigma}}\,,$$

in which the trace given by

$${\rm Tr}^*\ O \equiv {\rm Tr}\ \left[\begin{pmatrix} \mathbb{1}_{N\times N} & 0 \\ 0 & -\mathbb{1}_{M\times M}\end{pmatrix} O\right]$$

results from the periodicity of the worldsheet fermion superfield which was necessary to implement to the supersymmetric path ordering. By Witten's argument [15], only the zero modes contribute to the partition sum, giving [16, 3, 4]

$$\begin{aligned} S_{\rm RR} &= \tau_9 g_s \int C\wedge {\rm Tr}^*\ e^{2\pi\alpha' i\mathcal{F}}\,, \\ \mathcal{F} &= \begin{pmatrix} F^1 + iT^\dagger T & -i(DT)^\dagger \\ -iDT & F^2 + iTT^\dagger \end{pmatrix} \end{aligned} \tag{2.9}$$

$\mathcal{F}$ is the curvature of the superconnection, and as usual, the fermion zero modes form the basis for the dual vector space and all forms above are written with $\psi_0^\mu \to dx^\mu$. This expression is exact[1] and although it was derived for 2^{m-1} brane anti-brane pairs in [3, 4] it appears to have the correct properties for the general N brane M anti-brane case.

2.1 Covariant action

The above $\mathrm{D}\overline{\mathrm{D}}$ action may be further improved by making it manifestly covariant [5]. By requiring covariance of the action, it was possible to generalize the previous action to any tachyon profile. The main step in this improvement was to recognize that there is two independent U(1) and Lorentz invariants made of first derivatives in the tachyon fields.[2]

$$\mathcal{X} \equiv 2\pi\alpha'^2 g^{\mu\nu}\partial_\mu T\partial_\nu \overline{T}\,, \qquad \mathcal{Y} \equiv \left(2\pi\alpha'^2\right)^2 \left(g^{\mu\nu}\partial_\mu T\partial_\nu T\right)\left(g^{\alpha\beta}\partial_\alpha\overline{T}\partial_\beta\overline{T}\right). \tag{2.10}$$

With the normalizations chosen for convenience. For the linear profile $T = u_1x_1 + iu_2x_2$, the only translation invariant way to reexpress $u_{1,2}$ is as $u_{1,2} = \partial_{1,2}T^{1,2}$; then with $g^{\mu\nu} = \eta^{\mu\nu}$ we can calculate $\mathcal{X}$ and $\mathcal{Y}$,

$$\begin{aligned} \mathcal{X} &= 2\pi\alpha'^2(u_1^2 + u_2^2)\,, \\ \mathcal{Y} &= \left(2\pi\alpha'^2\right)^2 (u_1^2 - u_2^2)^2\,, \end{aligned}$$

[1] As discussed in [3], this action is exact in T and $A^\pm$ and their derivatives, but has corrections for non-constant RR forms.

[2] The metric is understood as being the open string metric $G^{\mu\nu} = \left(\frac{1}{G-B}\right)^{(\mu\nu)}$. Here, the antisymmetric part is set to zero.

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so the arguments of $\mathcal{F}$ in (2.6) can be written as

$$4\pi\alpha'^2 u_1^2 = \mathcal{X} + \sqrt{\mathcal{Y}}\,,$$
$$4\pi\alpha'^2 u_2^2 = \mathcal{X} - \sqrt{\mathcal{Y}}\,.$$

This provides a unique way to covariantize (2.6) as

$$S_{\mathrm{D}\overline{\mathrm{D}}} = -2\tau_9 \int d^{10}x\sqrt{-g}\; e^{-2\pi\alpha' T\overline{T}}\, \mathcal{F}(\mathcal{X}+\sqrt{\mathcal{Y}})\, \mathcal{F}(\mathcal{X}-\sqrt{\mathcal{Y}})\,. \tag{2.11}$$

Using symmetry constraint, one can also restore the gauge field coupling via the minimal coupling : $\partial_\mu T \to D_\mu T = \partial_\mu T + iA^-_\mu T$.

3. Multiple branes-anti-branes action

3.1 N D9-branes and M $\overline{\mathrm{D}}$9-branes

Let us consider the generalization of the action for $\mathrm{D}\overline{\mathrm{D}}$ to that of N D-branes and M anti-D-branes. We will then point out where only approximative results are possible or where a perturbative expansion should be performed (the path integral is not gaussian).

For a system of N D9-branes and M $\overline{\mathrm{D}}$9-branes (throughout we will assume $N \geq M$), the tachyon is that set of fields which stretches between the branes and the anti-branes; hence T is an $N \times M$ matrix, which transforms in the bifundamental of the $\mathrm{U}(N) \times \mathrm{U}(M)$ gauge group of the system. The potential for the tachyon fields can be derived from the work of [3, 4], who present the world-sheet sigma-model action for such systems, and evaluate the action for the tachyon when $N = M = 1$. The tachyon potential can be readily evaluated for general N and M, beginning with the world-sheet boundary insertion (2.2). Using the definition of supersymmetric path ordering as written in [3],

$$\begin{aligned}
\hat{P}\exp\left[\int d\hat{\tau}\mathcal{M}(\hat{\tau})\right] &= \sum_{N=0}^{\infty}\int d\hat{\tau}_1\cdots d\hat{\tau}_N \Theta(\hat{\tau}_{12})\Theta(\hat{\tau}_{23})\cdots\Theta(\hat{\tau}_{N-1,N})\mathcal{M}(\hat{\tau}_1)\cdots\mathcal{M}(\hat{\tau}_N)\\
&= 1 + \int d\tau\left(\mathcal{M}_1(\tau) - \mathcal{M}_0^2(\tau)\right) + \cdots\\
&= P\exp\left[\int d\tau\left(\mathcal{M}_1(\tau) - \mathcal{M}_0^2(\tau)\right)\right]
\end{aligned} \tag{3.1}$$

in which $\hat{\tau}_{12} = \tau_1 - \tau_2 + \theta_1\theta_2$, P in the result is normal path ordering after integration over superspace, and $\mathcal{M}_{0,1}$ are the parts of the matrix $\mathcal{M}$ which are proportional to zero and one power of θ.

For constant tachyons, and vanishing gauge fields,

$$\mathcal{M}_0 = \sqrt{\alpha'}\begin{pmatrix} 0 & T^\dagger \\ T & 0 \end{pmatrix}, \qquad \mathcal{M}_1 = 0\,,$$

and the path ordering is irrelevant, so the potential for a general numbers of branes and anti-branes is simply

$$
\begin{aligned}
V(T,T^\dagger) &= \tau_9 \mathrm{Tr}\exp\left[-2\pi\alpha'\begin{pmatrix} T^\dagger T & 0 \\ 0 & TT^\dagger\end{pmatrix}\right] \\
&= \tau_9\left[(N-M) + 2\mathrm{Tr}e^{-2\pi\alpha' TT^\dagger}\right]. \qquad (3.2)
\end{aligned}
$$

This potential clearly agrees with all the physics expected from Sen's arguments [17]; when $T=0$, since the branes and anti-branes are frozen at the unstable maximum, the potential is just the tension of $(N+M)$ D9 branes. Also, it is known that the M anti-branes should decay with M of the branes, leaving only $(N-M)$ D9-branes; at its stable minimum, $|T|\to\infty$, the potential takes the value $\tau_9(N-M)$, which is the tension of the remaining branes.

For non-constant tachyon, the path integral is not gaussian and cannot be performed in general. Nevertheless we might be able to describe some parts of this system approximatively. The calculations are quite involved for the generic case so, for the rest of this paper, we will focus on 2 D9-branes and 1 $\overline{\mathrm{D}}$9-brane.

3.2 Two D9-branes and 1 $\overline{\mathrm{D}}$9-brane

For the case of 2 D9-branes and 1 $\overline{\mathrm{D}}$9-brane, the boundary insertion is a 3×3 matrix. The boundary fermions method use in [3] to perform the path ordering is applicable for a matrix that can be expanded in terms of SO$(2m)$ generators. This is not the case here. One way to circumvent this problem is to extend the matrix to a 4×4 matrix, set the extra tachyons to zero and then use boundary fermions as before. We then get a non-gaussian path integral which can only be done perturbatively. Another way is to do the path ordering by hand term by term in the expansion series. This way turns out to be useful to see the behavior of the action and to see what kind of approximation we will need to do.

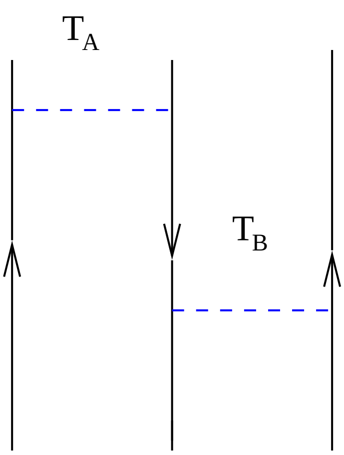

Figure 1: Two D-branes (arrow up) and 1 anti-D-brane (arrow down) with two complex tachyons stretching between them. The branes are all sitting on top of each other.

We start with the supersymmetrically ordered expression (3.1) which is valid for any boundary insertion of the form (2.2). We will set the gauge field and the Kalb-Ramond fields to 0 but we otherwise consider a general case of 2 D9-branes and 1 $\overline{\mathrm{D}}$9-brane. The boundary insertion is then (after integrating over θ):

$$
\begin{aligned}
M_1(\tau) - M_0^2(\tau) &= M(\tau) = \alpha'\begin{pmatrix} -T^\dagger T & \sqrt{\alpha'}\psi^\mu\partial_\mu T^\dagger \\ \sqrt{\alpha'}\psi^\mu\partial_\mu T & -TT^\dagger\end{pmatrix} \\
T &= \begin{pmatrix} T_A & T_B\end{pmatrix} \qquad T^\dagger = \begin{pmatrix}\overline{T}_A \\ \overline{T}_B\end{pmatrix} \\
T_A &= T_1 + iT_2 \qquad T_B = T_3 + iT_4\,, \qquad (3.3)
\end{aligned}
$$

where T_A and T_B are the two complex fields between the two branes and the anti-brane as shown in figure 1. $T_1\ldots T_4$ are 4 real tachyon fields.

The path ordering can be done by hand:

$$
\begin{aligned}
e^{-S_{\partial\Sigma}} &= \mathrm{Tr}\hat{P}\exp\left[\int d\tau \mathcal{M}(\tau)\right] \\
&= \tau_9 \mathrm{Tr}\sum_{N=0}^{\infty}\int d\tau_1 \ldots d\tau_N \Theta_{1,2}\ldots\Theta_{N-1,N} M(\tau_1)\ldots M(\tau_N) \\
&= \tau_9 \mathrm{Tr}\left(\mathbb{1} + \int d\tau_1 M(\tau_1) + \int d\tau_1 d\tau_2 M(\tau_1)M(\tau_2)\Theta_{12} + \ldots\right)
\end{aligned} \tag{3.4}
$$

where $\Theta_{12} = \Theta(\tau_1 - \tau_2)$. It is simple to see (by working out the first few orders) that the potential part can be exponentiated.

$$
\begin{aligned}
e^{-S_{\partial\Sigma}} = \tau_9 \Bigg(& V(T, T^\dagger) + \alpha'^2 \int d\tau_1 d\tau_2 \mathrm{Tr}(\psi^\mu \partial_\mu T_1 \psi^\nu \partial_\nu T_2^\dagger)\mathrm{sgn}(\tau_{12}) - \\
& - \alpha'^3 \int d\tau_1 d\tau_2 d\tau_3 \mathrm{Tr}(\psi^\mu \partial_\mu T_2 (T^\dagger T)_1 \psi^\nu \partial_\nu T_3^\dagger)(-\Theta_{32}\Theta_{21} - \Theta_{13}\Theta_{32} + \Theta_{21}\Theta_{13}) - \\
& - \alpha'^3 \int d\tau_1 d\tau_2 d\tau_3 \mathrm{Tr}((TT^\dagger)_1 \psi^\mu \partial_\mu T_2 \psi^\nu \partial_\nu T_3^\dagger)(-\Theta_{31}\Theta_{12} + \Theta_{12}\Theta_{23} + \Theta_{23}\Theta_{31})\Bigg) + \\
& + \mathcal{O}(\alpha'^4),
\end{aligned} \tag{3.5}
$$

where $V(T, T^\dagger)$ is given in (3.2). The $\mathrm{sgn}(\tau_{12})$ comes from a factor of $\Theta_{12} - \Theta_{21}$ and it reflects the non-trivial ordering process involved in this path integral. Putting one complex tachyon to zero, the two expressions at third order are the same ($T^\dagger T = TT^\dagger = T\overline{T}$) and the various factors of Θ combine in pairs to give 1 if the τ are cyclically ordered or -1 if not. In this case, one can then see the pattern and the kinetic term exponentiate (together with the potential) to give the D$\overline{\text{D}}$ boundary insertion:

$$
e^{-S_{\partial\Sigma}}|_{\mathrm{D}\overline{\mathrm{D}}} = 2\tau_9 e^{-\alpha' \int d\tau \left(TT^\dagger + \alpha' \psi^\mu \partial_\mu T \frac{1}{\partial_\tau} \psi^\nu \partial_\nu T^\dagger\right)} .
$$

In the DD$\overline{\text{D}}$ case, the two terms at third order are different and the kinetic part does not exponentiate any longer. We have:

$$
\begin{aligned}
\psi^\mu \partial_\mu T (T^\dagger T) \psi^\nu \partial_\nu T^\dagger &= T_A \overline{T}_A \psi^\mu \partial_\mu T_A \psi^\nu \partial_\nu \overline{T}_A + T_B \overline{T}_B \psi^\mu \partial_\mu T_B \psi^\nu \partial_\nu \overline{T}_B + \\
&\quad + T_A \overline{T}_B \psi^\mu \partial_\mu T_A \psi^\nu \partial_\nu \overline{T}_B + T_B \overline{T}_A \psi^\mu \partial_\mu T_B \psi^\nu \partial_\nu \overline{T}_A
\end{aligned} \tag{3.6}
$$

$$
(TT^\dagger)\psi^\mu \partial_\mu T \psi^\nu \partial_\nu T^\dagger = (T_A \overline{T}_A + T_B \overline{T}_B)(\psi^\mu \partial_\mu T_A \psi^\nu \partial_\nu \overline{T}_A + \psi^\mu \partial_\mu T_B \psi^\nu \partial_\nu \overline{T}_B) . \tag{3.7}
$$

These two expressions differ by various mixing terms between T_A and T_B. So the path integral cannot be carried out in general. Let us take the approximation where these mixing terms are ignored. The task for finding an effective action is then greatly simplified. As we shall see, this approximate effective action in closed form is still quite useful in capturing some interesting physics. This approximation physically signifies that our action will only be valid when 1 tachyon is frozen while the other rolls. We will see in section §4 how we can physically achieve such a situation using t-duality.

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3.3 The DD$\overline{\text{D}}$ action

Ignoring the mixing terms, the kinetic part will exponentiate like before. Requiring the U(2) symmetry in the path integral, we obtain, for the boundary insertion in the path integral ($TT^\dagger = T_A\overline{T}_A + T_B\overline{T}_B$) :

$$e^{-S_{\partial\Sigma}} = \tau_9 \left(1 + 2e^{-\alpha' \int d\tau \left(TT^\dagger + \alpha' \psi^\mu \partial_\mu T \frac{1}{\partial_\tau} \psi^\nu \partial_\nu T^\dagger\right)}\right), \tag{3.8}$$

where the tachyon is now a SU(2) doublet. Although the mixing terms are important in some situations, they are absent in some dynamical situations. For example, in the simple tachyon-rolling (that is, without forming defects), all except one real tachyon should remain zero. In this case, the mixing terms are absent. Tachyon rolling that involves the formation of a codimension-2 (unstable) vortex involves only a single complex tachyon, so the mixing terms are absent again. Mindful of this approximation, let us move on.

Consider a linear profile $T_i = u_i X^i$, $i = 1, 2, 3, 4$. This path integral (the usual disk action and the previous boundary insertion (3.8)) is gaussian and exactly solvable. One get (in flat spacetime), with $c_i = u_i^2$:

$$S_{(\text{DD}\overline{\text{D}})_9} = -\tau_9 \int d^{10}x \left(1 + 2e^{-2\pi\alpha' c_i x_i^2} \prod_{i=1}^{4} \mathcal{F}\left(\frac{(2\pi\alpha')^2}{\pi} c_i\right)\right). \tag{3.9}$$

For a more general linear tachyon profile, $T = UX$ and U is a 4×4 matrix (T and X are 4-vectors for the 4 real tachyons). Going to the diagonal basis for $U^\dagger U$ and let c_i be the eigenvalues of $U^\dagger U$, we have $TT^\dagger = c_i x_i^2$. Again, we obtain (3.9).

We can generalize the above action to N D-branes and 1 anti-D-brane. The tachyon would be in the fundamental representation of SU(N) and we therefore will have N complex tachyons and 2N $\mathcal{F}$ in the action. The approximation is again that while 1 tachyon is rolling all the other tachyons must be frozen.

3.4 U(2) $\times$ U(1) covariant action

As it is, our action (3.9) is not covariant under U(2) $\times$ U(1) since we have to choose a particular profile to be able to solve the path integral. It would be interesting to covariantize this action, that is, replace the c_i by gauge and Lorentz invariants involving only first derivatives of T. We can then replace the derivatives by covariant derivatives. This turns out to be somewhat complicated, so we leave the details of the derivation in the appendix. Here we give the main result. In the linear tachyon profile, we have $T = UX$, where U is a 4×4 matrix. A general gauge and spacetime rotation can reduce the number of entries in U from 16 to 6. Let us call them u_i, $i = 1, 2, .., 6$. So we can express the 4 eigenvalues c_i in terms of the 6 u_i. Next we find that there are 6 independent gauge and Lorentz invariants that involve only first derivatives of T. This set involve invariants up to 8 derivatives. Since we can write the u_i in terms of the 6 invariants, the c_i can also be written in terms of those invariants. This allows us to express the DD$\overline{\text{D}}$ action in terms of the 6 invariants. The dependence of other invariants (with higher power of first derivatives) in terms of a given basis is somewhat non-linear, so there is no obvious choice of basis. A relatively simple choice of basis gives

$$
\begin{aligned}
4\pi\alpha'^2 c_1 &= \mathcal{X}_+ + \sqrt{\mathcal{Y}_+} \\
4\pi\alpha'^2 c_2 &= \mathcal{X}_+ - \sqrt{\mathcal{Y}_+} \\
4\pi\alpha'^2 c_3 &= \mathcal{X}_- + \sqrt{\mathcal{Y}_-} \\
4\pi\alpha'^2 c_4 &= \mathcal{X}_- - \sqrt{\mathcal{Y}_-} \\
\mathcal{X}_\pm &\equiv 2\pi\alpha'^2 \left(K_1 \pm \sqrt{K_3}\right) \qquad \mathcal{Y}_\pm \equiv (2\pi\alpha'^2)^2 \left(K_2 + K_4 \pm \sqrt{4K_2K_4 - K_5}\right) \\
K_1 &= \frac{1}{2}\partial_\mu T_i \partial^\mu \overline{T}^i \qquad\qquad (3.10) \\
K_2 &= \frac{1}{4}\left(\partial_\mu T_i \partial^\mu \overline{T}^j \partial_\nu T_j \partial^\nu \overline{T}^i - \epsilon_{ij}\epsilon_{mn}\partial_\mu T_i \partial^\mu \overline{T}_m \partial_\nu T_j \partial^\nu \overline{T}_n\right) \\
K_3 &= \frac{1}{4}\left(\partial_\mu T_i \partial^\mu T_j \partial_\nu \overline{T}^j \partial^\nu \overline{T}^i + \sqrt{\epsilon_{ij}\epsilon_{kl}\epsilon_{mn}\epsilon_{ab}\partial_\mu T_i \partial^\mu T_k \partial_\nu \overline{T}_m \partial^\nu \overline{T}_a \partial_\delta T_j \partial^\delta T_l \partial_\kappa \overline{T}_n \partial^\kappa \overline{T}_b}\right) \\
K_4 &= \frac{1}{4}\left(\partial_\mu T_i \partial^\mu T_j \partial_\nu \overline{T}^j \partial^\nu \overline{T}^i - \sqrt{\epsilon_{ij}\epsilon_{kl}\epsilon_{mn}\epsilon_{ab}\partial_\mu T_i \partial^\mu T_k \partial_\nu \overline{T}_m \partial^\nu \overline{T}_a \partial_\delta T_j \partial^\delta T_l \partial_\kappa \overline{T}_n \partial^\kappa \overline{T}_b}\right) \\
K_5 &= \partial_\mu T_i \partial^\mu T_j \partial_\nu \overline{T}^j \partial^\nu T_k \partial_\delta \overline{T}^k \partial^\delta T_m \partial_\kappa \overline{T}^m \partial^\kappa \overline{T}^i - \\
&\quad - \partial_\mu T_i \partial^\mu T_j \partial_\nu \overline{T}^j \partial^\nu T_k \partial_\delta \overline{T}^k \partial^\delta \overline{T}^m \partial_\kappa T_m \partial^\kappa \overline{T}^i \,,
\end{aligned}
$$

where Greek indices refers to Lorentz indices and latin indices are SU(2) indices ($i = A, B$). Now the action can be written in a covariant way:

$$
\begin{aligned}
S_{\mathrm{DD\overline{D}}} = -\tau_9 \int d^{10}x\sqrt{-g}\Big(1 + 2e^{-2\pi\alpha' T^\dagger T}\, \mathcal{F}\left(\mathcal{X}^+ + \sqrt{\mathcal{Y}^+}\right) \mathcal{F}\left(\mathcal{X}^+ - \sqrt{\mathcal{Y}^+}\right) \times \\
\times \mathcal{F}\left(\mathcal{X}^- + \sqrt{\mathcal{Y}^-}\right) \mathcal{F}\left(\mathcal{X}^- - \sqrt{\mathcal{Y}^-}\right)\Big) .
\end{aligned} \qquad (3.11)
$$

As is shown at the end of the appendix, the U(1) and Lorentz invariants of the $\mathrm{D\overline{D}}$ system arise naturally by setting $T_B = 0$.

To T-dualize the above action such that we can move one brane away from the pair, we need to covariantize further by restoring the gauge field. The gauge group is $\mathrm{U}(2)\times\mathrm{U}(1) = \mathrm{SU}(2)\times\mathrm{U}(1)_-\times\mathrm{U}(1)_+$ and, as usual, the tachyon does not couple to the $\mathrm{U}(1)_+$. Therefore,

To restore $\mathrm{SU}(2)\times\mathrm{U}(1)_-$ we need to change the derivatives to covariant derivatives:

$$
\partial T \to D_\mu T = \partial_\mu T + i\left(A^a_\mu \frac{\sigma_a}{2} + \phi^-_\mu\right) T \,.
$$

It turns out to be impossible to restore the gauge kinetic term simply. The simplest natural way would be a DBI-kind of prefactor[3] in front of the whole action but one needs to make a difference between the gauge field living on each of the two branes. It must be that the gauge kinetic term mixes with the tachyon in a non-trivial manner.

Therefore, our $\mathrm{SU}(2)\times\mathrm{U}(1)$ invariant without the gauge field kinetic term is:

$$
S_{(\mathrm{DD\overline{D}})_9} = -\tau_9 \int d^{10}x \left(\sqrt{-g}\left(1 + 2e^{-2\pi\alpha' T T^\dagger}\, \mathcal{F}^4\right)\right) . \qquad (3.12)
$$

Where the arguments of $\mathcal{F}$ have been omitted and contain covariant derivatives.

[3]We would need to use the symmetric trace like was done in [9].

3.5 The RR coupling

Due to its topological nature, the RR action (2.9) should be exact for any number of branes and anti-branes [16, 3, 4]. For $\mathrm{DD\overline{D}}$ action, the superconnection is given by:

$$\mathcal{F} = \begin{pmatrix} F^{(1)} + iT^{\dagger}T & -i(DT)^{\dagger} \\ -iDT & F^{(2)} + iTT^{\dagger} \end{pmatrix} \tag{3.13}$$

where F^1 and $T^{\dagger}T$ are 2×2 matrices. We use a shorthand notation $\mathcal{T} = T^{\dagger}T$ and $t = TT^{\dagger}$.

$$\begin{aligned} F^1 &= \begin{pmatrix} F_{11} & F_{12} \\ F_{21} & F_{22} \end{pmatrix} \\ \mathcal{T} \equiv T^{\dagger}T &= \begin{pmatrix} T_A\overline{T}_A & T_A\overline{T}_B \\ T_B\overline{T}_A & T_B\overline{T}_B \end{pmatrix} \\ t \equiv TT^{\dagger} &= T_A\overline{T}_A + T_B\overline{T}_B \,. \end{aligned}$$

For example, one can work out the coupling to C_8 for the case of 2 D-branes and 1 anti-D-brane. We just expand the exponential (2.9) keeping every 2-forms. After some manipulations where the following identity is used,

$$\mathrm{Tr}\begin{pmatrix} -\mathcal{T} & \\ & -t \end{pmatrix}^n = (-1)^n \mathrm{Tr}\begin{pmatrix} \mathcal{T}^n & \\ & t^n \end{pmatrix} = (-1)^n t^{n-1} \mathrm{Tr}\begin{pmatrix} \mathcal{T} & \\ & t \end{pmatrix}$$

we get($\lambda = 2\pi\alpha'$):

$$\begin{aligned} S_{RR}|_{C_8} = \mu_9 \int (-iC_8) \wedge \Bigg(& \frac{\lambda}{t}(e^{-\lambda t} - 1) DT \wedge DT^{\dagger} - \\ & - \frac{\lambda}{t}\left(e^{-\lambda t}\left(\lambda + \frac{1}{t}\right) - \frac{1}{t}\right) DT\mathcal{T}DT^{\dagger} + i\lambda(F_{11} + F_{22} - F^{(2)}) + \\ & + i\frac{\lambda}{t}\left(e^{-\lambda t} - 1\right)\left(\mathrm{Tr}\left(\mathcal{T}F^{(1)}\right) - tF^{(2)}\right)\Bigg) \,. \end{aligned} \tag{3.14}$$

4. T-duality of the $\mathrm{DD\overline{D}}$ effective action

Our action is only valid when one tachyon rolls while the other is frozen. This scenario can be physically realized by moving one brane away, such that T_B is no longer tachyonic. To do so, we have to consider Dp-branes where $p < 9$. This can be done using T-duality and we give the relevant formulas here.

The T-dual properties of the various fields in the action are well known; the gauge fields in the T-dual directions transform into the adjoint scalars, the metric and Kalb-Ramond field mix, the string coupling scales and finally the field strength in the T-dual direction gives rise to a commutator. Being an open string scalar state, the tachyon is inert under T-duality. Under T-duality in directions labeled by uppercase Latin indices, (lowercase

Latin indices labeling unaffected directions on the brane), the fields transform as [19]

$$
\begin{array}{lll}
T \to T\,, & A_a \to A_a\,, & A_I \to \dfrac{\Phi^I}{2\pi\alpha'}\,, \\
E_{\mu\nu} \equiv g_{\mu\nu} + B_{\mu\nu}, & e^{2\phi} \to e^{2\phi}\det E^{IJ}\,, & E_{IJ} \to E^{IJ} \\
E_{ab} \to E_{ab} - E_{aI}E^{IJ}E_{Jb}\,, & E_{aI} \to E_{aK}E^{KI}\,, & E_{Jb} \to -E^{JK}E_{Kb}\,, \\
F_{aI} \to \dfrac{D_a\Phi^I}{2\pi\alpha'}\,, & F_{IJ} \to \dfrac{i[\Phi^I,\Phi^J]}{4\pi^2\alpha'^2}\,, &
\end{array}
\tag{4.1}
$$

where E^{IJ} is the matrix inverse to E_{IJ} and it can therefore be used to lower or raise indices. We used normal coordinates where the fields are independent of the coordinates we are 'dualing'. The non-commutative aspect of the system appears in the T-dual of the field strength where we get the commutator of $\mathrm{U}(2)\times\mathrm{U}(1)$ matrix valued scalar fields. To treat this we follow [19] and introduce

$$
Q^I_J = \delta^I_J + i2\pi\alpha'[\Phi^I,\Phi^K]E_{KJ}\,. \tag{4.2}
$$

The result of T-dualing $9-p$ dimensions can be written most simply by defining the pull-back in normal coordinates as:

$$
\begin{aligned}
P[E_{ab}] &\equiv E_{ab} + E_{aI}\partial_b\Phi^I + \partial_a\Phi^I E_{Ib} + \left(\partial_a\Phi^I E_{IJ}\partial_b\Phi^J\right), \\
P[E_{aI}] &\equiv E_{aI} + \partial_a\Phi^J E_{JI} \\
P[E_{Jb}] &\equiv E_{Jb} + E_{JI}\partial_b\Phi^I\,.
\end{aligned}
\tag{4.3}
$$

where the scalar Φ^I are:

$$
\Phi^I = \begin{pmatrix} \Phi^{I\,1} & \\ & \Phi^{I\,2} \end{pmatrix} = \begin{pmatrix} \Phi^I_{11} & \Phi^I_{12} & 0 \\ \Phi^I_{21} & \Phi^I_{22} & 0 \\ 0 & 0 & \Phi^{I\,2} \end{pmatrix}.
$$

We can separate $\Phi^{I\,1}$ into a SU(2) part $W^{I\,1}$ and a U(1) trace $\phi^{I\,1}$. The difference $\psi^I \equiv \phi^{I\,1} - \Phi^{I\,2}$ is the scalar representing the (DD - $\overline{\mathrm{D}}$)$_p$ separation. The tachyon couples to $\varphi^I = \Phi^{I\,1} - \Phi^{I\,2} = W^I + \psi^I$.

$$
D^I T = \partial^I T + \frac{i}{2\pi\alpha'}\varphi^I T\,. \tag{4.4}
$$

In calculating the pull-back of any quantity only the indices corresponding to the directions along the brane are affected. After T-dualing the fields in (3.12) [4] as above and performing manipulations similar to those in [19], we obtain the improved action for 2 D$_p$-branes and 1 anti-D$_p$-brane:

$$
\begin{aligned}
S_{(\mathrm{DD}\overline{\mathrm{D}})_p} = -\tau_p \int d^{p+1}x\sqrt{-\det[G]}\Big(1 + e^{-2\pi\alpha' TT^\dagger}\,\mathcal{F}\left(\mathcal{X}^+ + \sqrt{\mathcal{Y}^+}\right)\mathcal{F}\left(\mathcal{X}^+ - \sqrt{\mathcal{Y}^+}\right) \times \\
\times\,\mathcal{F}\left(\mathcal{X}^- + \sqrt{\mathcal{Y}^-}\right)\mathcal{F}\left(\mathcal{X}^- - \sqrt{\mathcal{Y}^-}\right)\Big),
\end{aligned}
\tag{4.5}
$$

[4] It is important to realise that equation (3.12) does not have a DBI prefactor and T-duality will generate one naturally by mixing the metric and the field strenght. Like we said previously, it is inconsistent to have such a global prefactor in front of our action because it does not make the difference between the two D-branes. For our sake, this is unimportant since we only want to show how the tachyon gets a mass. One should not trust the DBI prefactor but the covariant devrivatives are still under control and give sensible results.

where now the effective metric contains the spacetime metric pulled-back to the brane worldvolume (and includes any non-zero NS-NS B field) and has corrections from the non-commutativity of the coordinates.

$$G_{ab} \equiv P[E_{ab} + E_{aI}(Q^{-1}-\delta)^{IJ}E_{Jb}] + 2\pi\alpha' F_{ab}\,.$$

The covariant derivative dependence of $\mathcal{X}_\pm$ and $\mathcal{Y}_\pm$ in (3.10) leads to Φ dependence in the T-dual action. Instead of giving the complete expressions for $\mathcal{X}_\pm$ and $\mathcal{Y}_\pm$ (they are rather long), we will show how to get all the essential features.

The building blocks of all the invariants are $\partial_\mu \overline{T}_i G^{\mu\nu}\partial_\nu T^i$ and two others contracted as tensor ((TT) and $(\overline{TT})$). It is important to note that the SU(2) indices are not affected by T-duality since they just refer to different scalars. To get the T-duality of $\partial_\mu \overline{T}_i G^{\mu\nu}\partial_\nu T^i$, we just need to compute how the inverse of the metric transforms and how derivatives transform.

$$\begin{aligned}
[G_{\mu\nu}]^{-1} &= \begin{pmatrix} G^{ba} & -G^{ba}P[E'_{aI}] \\ P[E'_{Jb}]G^{ba} & (E'_{JI} - P[E'_{Jb}]G^{ba}P[E'_{aI}]) \end{pmatrix} \\
E'_{aI} &\equiv E_{aJ}(Q^{-1})^J_I \\
E'_{Jb} &\equiv (Q^{-1})^K_J E_{Kb} \\
E'_{JI} &\equiv E_{JL}(Q^{-1})^L_I\,.
\end{aligned} \tag{4.6}$$

We can now compute the transformation of our simplest invariant (K_1) from the last equation. It is completely similar to what was done in [5] but we need to insert factors of Q^{-1} in the pullbacks.

$$\partial_\mu \overline{T}_i G^{\mu\nu}\partial_\nu T^i = \mathrm{Re}\left[\begin{array}{r} G^{ab}_{1,2}D_a T^\dagger D_b T + \frac{1}{(2\pi\alpha')^2}\left(E'_{IJ} - P[E'_{Ib}]G^{ba}P[E'_{aJ}]\right)T^\dagger\varphi^{\dagger I}\varphi^J T \\ -\frac{i}{2\pi\alpha'}\left(P[E'_{Ia}]G^{ab}T^\dagger\varphi^{\dagger I}D_b T + G^{ab}P[E'_{bI}]D_a T^\dagger\varphi^I T\right)\end{array}\right]. \tag{4.7}$$

These expressions simplify considerably in Minkowski spacetime when $B=0$ and $A^{1,2}=0$:

$$\begin{aligned}
G_{ab} \quad &= \quad \eta_{ab} + (Q^{-1})_{IJ}\left(\partial_a\Phi^I\partial_b\Phi^J\right) \\
\partial_\mu \overline{T}_i G^{\mu\nu}\partial_\nu T^i \quad &\xrightarrow[B=0,\ A^{1,2}=0]{g=\eta} \quad \Big[G^{ab}\partial_a T^\dagger\partial_b T - \frac{i}{2\pi\alpha'}\times \\
&\times\Big((Q^{-1})_{IJ}\partial_a\Phi^J G^{ab}T^\dagger\varphi^{\dagger I}\partial_b T + G^{ab}\partial_b\Phi^J(Q^{-1})_{JI}\partial_a T^\dagger\varphi^I T\Big) + \\
&+\frac{1}{(2\pi\alpha')^2}\Big((Q^{-1})_{IJ} - (Q)^{-1}_{IK}\partial_b\Phi^K G^{ab}\partial_a\Phi^M(Q)^{-1}_{MJ}\Big)\times \\
&\times T^\dagger\varphi^{\dagger I}\varphi^J T\Big]\,.
\end{aligned} \tag{4.8}$$

The others invariants can be built the same way. The last expression is somewhat complicated because we kept every pieces. It contains non-commutative effects which might be interesting to study. In this paper, we just want to set-up the formalism and show explicitly how the tachyon gets a mass when we move one brane away. Suppose we have 2 D_8-branes and 1 anti-D_8-brane that are transverse to the X_1 direction, at $X_1 = 0$.

Number of rolling tachyon	Tension of solitons	Action after condensation
1	$2\tau_9\sqrt{2\pi^2\alpha'} = \sqrt{2}\tau_8$	$\int d^{10}x \left(\tau_9 + \sqrt{2}\tau_8\delta(x_{10})e^{-2\pi\alpha' T_i\overline{T}^i}\,\mathcal{F}^3\right)$
2	$2\tau_9(2\pi^2\alpha') = \tau_7$	$\int d^{10}x \left(\tau_9 + \tau_7\delta(x_{10},x_9)e^{-2\pi\alpha' T_i\overline{T}^i}\,\mathcal{F}^2\right)$
3	$2\tau_9(2\pi^2\alpha')^{\frac{3}{2}} = \frac{1}{\sqrt{2}}\tau_6$	$\int d^{10}x \left(\tau_9 + \frac{1}{\sqrt{2}}\tau_6\delta(x_{10},x_9,x_8)e^{-2\pi\alpha' T_i\overline{T}^i}\,\mathcal{F}\right)$
4	$2\tau_9(2\pi^2\alpha')^2 = \frac{1}{2}\tau_5$	$\int d^{10}x \left(\tau_9 + \frac{1}{2}\tau_5\delta(x_{10},x_9,x_8,x_7)\right)$

Table 1: Resulting actions and tensions for linear tachyonic profile.

Let us move one of the D-branes to the position Φ_{11} while the remaining D$\overline{\text{D}}$-pair is held fixed at $\Phi^2 = \Phi_{22} = 0$. The matrix Φ is therefore:

$$\Phi \simeq \begin{pmatrix} \Phi_{11} & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}.$$

To simplify, we consider stationary branes : $\partial\Phi = 0$. This yields a much simpler kinetic term:

$$\partial_\mu \overline{T}_i G^{\mu\nu}\partial_\nu T^i = \eta^{ab}\partial_a T \partial_b T^\dagger + \frac{1}{(2\pi\alpha')^2} T_A \Phi^*_{11}\Phi_{11}\overline{T}_A\,. \tag{4.9}$$

In this case, T_A becomes massive and only T_B remains tachyonic.

5. Some properties of the DD$\overline{\text{D}}$ action

The formation of defects in a brane system has been studied using K-theory arguments [13, 20]. We expect that the tachyon condensation in the DD$\overline{\text{D}}$ system should produce only non-BPS defects. A simple example is a codimension-2 defect, i.e., a vortex, produced by the annihilation of D$\overline{\text{D}}$ pair. Such a vortex is unstable and it will eventually dissolve into the remaining D-brane.

If we take the action (3.11) and use it for the case where all the branes are sitting on top of each other, we do not get any stable defects. As one can see from table 1, we get the right tension for, the non-BPS, codimension 1 brane and for the vortex (codimension 2) but both of them are unstable due to the presence of the remaining tachyon. Instantons (codimension 4) would be stable but we do not get the right tension. In fact, it is important to understand that the last two lines of table 1 do not represent any actual decay products of the tachyons. The mixing terms come in and it is impossible to neglect them any longer. In the following table the first two lines represent possible unstable defects. The last two lines, on the other hand, give unphysical defects and are outside the region of validity of our approximation.

Solitons can be obtained with a linear profile for the tachyon $T_i = u_i x_i$ in the limit where u_i goes to infinity. To get the tension of the solitons, we just have to put this profile into the action and integrate out the space over which the tachyon condenses (using the large x limit of $\mathcal{F}(x)$).

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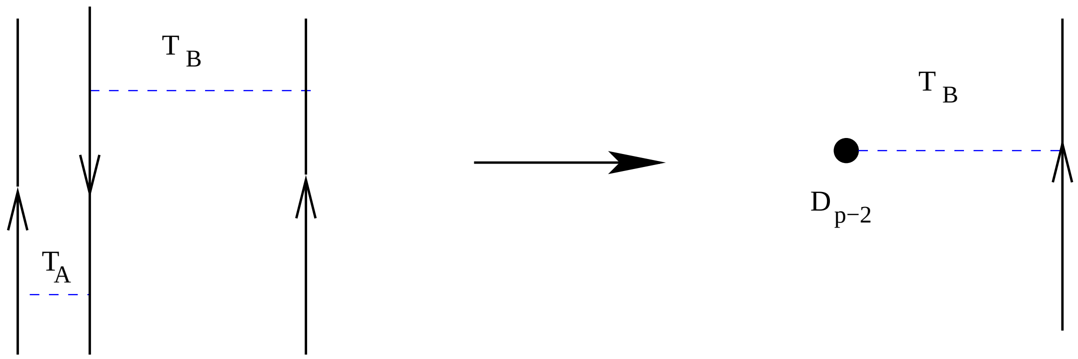

Figure 2: A stable vortex is formed if one brane is away from the D$\overline{\rm D}$ pair such that T_B is massive. As the distance is reduced to the point where the T_B becomes tachyonic, the vortex will dissolve.

The vortex is an interesting case since the action we get might be able to describe the dissolution of a D_{p-2} into a D_p. This is the T-dual analog of the recombination of strings [21]. We believe that the following action, with addition of a gauge field coupling, might be able to describe (at least in a qualitative level) the dissolution of branes

$$S_{D_9D_7} = -\tau_9 \int d^{10}x\sqrt{-g} - \tau_7 \int d^8x\sqrt{-g}e^{-2\pi\alpha' T_B\overline{T}_B}\, \mathcal{F}(\mathcal{X}+\sqrt{\mathcal{Y}})\,\mathcal{F}(\mathcal{X}-\sqrt{\mathcal{Y}})\,, \qquad (5.1)$$

where the invariants are the same as in [5]:

$$\mathcal{X} \equiv 2\pi{\alpha'}^2 g^{\mu\nu}\partial_\mu T_B\partial_\nu\overline{T}_B\,, \qquad \mathcal{Y} \equiv \left(2\pi{\alpha'}^2\right)^2\left(g^{\mu\nu}\partial_\mu T_B\partial_\nu T_B\right)\left(g^{\alpha\beta}\partial_\alpha\overline{T}_B\partial_\beta\overline{T}_B\right),$$

At first sight it might seems that the two parts of the action (5.1) are completely disconnected. This is true as long as we only have the tachyon field. In order to describe the decay (dissolution), we need to restore the gauge fields. We will study this issue elsewhere. One can get stable vortices in our system if we take one brane away. In this case, only T_A of the D$\overline{\rm D}$ pair is present and the tachyon rolling simply gives the multi-vortex solitons of the D$\overline{\rm D}$ system [5]. We can also include gauge flux inside the vortex.

At the classical open string level, the brane separation is a modulus. In the above case, T_B has a real positive mass, so it is frozen at $T_B = 0$, and the RR coupling with D_{p-2}-branes simplifies considerably.

$$S_{RR}|_{C_8} = -\frac{\tau_{p-2}g_s\lambda}{2\pi}\int(-iC_{p-1})\wedge\left(-\frac{i}{\lambda}F_{22}+e^{-\lambda T_A T_A^\dagger}(DT_A\wedge DT_A^\dagger-\frac{i}{\lambda}(F_{22}-F^{(2)}))\right). \quad (5.2)$$

The coefficient in front comes from the relation between the tension of D_p and D_{p-2}: $\mu_p\lambda = \tau_p g_s\lambda = \frac{\tau_{p-2}g_s}{2\pi}$. It gives the correct RR charge for a D_{p-2}-brane.

6. Concluding remarks

In this paper, we attempt to generalize the D$\overline{\rm D}$ effective action to that of the DD$\overline{\rm D}$ effective action in boundary superstring field theory. Besides the non-abelian properties that are intrinsic in the DD$\overline{\rm D}$ action, we find that the covariantization procedure also becomes quite complicated. Taking the approximation where one tachyon is frozen while the other rolls (which amount to neglecting all the cross-term between the two tachyons), we obtain a $\mathrm{DD\overline{D}}$ action in closed form. We use it to construct the D_pD_{p-2} action. The D_pD_{p-2} system may provide an interesting brane inflationary model and will be studied in future work.

Another interesting application of this $\mathrm{DD\overline{D}}$ system is to study how the system decay. It is clear that a $\mathrm{D\overline{D}}$ pair in the $\mathrm{DD\overline{D}}$ system will annihilate, leaving behind a single D-brane. Where does the energy go? In the pure $\mathrm{D\overline{D}}$ system, the decay releases energy to some combination of defects, closed string modes and tachyon matter. In the $\mathrm{DD\overline{D}}$ system, the tachyons couple to U(1) gauge fields of the remaining brane. So, as the tachyon rolls, energy will also go to the open string modes which are absent in the pure $\mathrm{D\overline{D}}$ system. If we treat the $\mathrm{DD\overline{D}}$ system as an inflationary model, energy that goes to open string modes allow the heating of the universe at the end of inflation. Since big bang cosmology puts strong bounds on the production of defects, closed string modes and tachyon matter, it will be important to determine the end products of tachyon rolling as a theoretical test of brane inflation.

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A. $\mathrm{SU}(2) \times \mathrm{U}(1)$ invariants

The gauge group living on a system of coincident M D-branes and $\overline{M}$ anti-D-branes is just $\mathrm{U}(M) \times \mathrm{U}(\overline{M})$. The tachyon field transforms as a bifundamental. We are interested in the case where $M = 2$ and $\overline{M} = 1$. The gauge symmetry is therefore $\mathrm{U}(2) \times \mathrm{U}(1)$. Since T is charged only under a linear combination of the U(1)s, we need consider only $\mathrm{SU}(2) \times \mathrm{U}(1)$. We want to find a basis of gauge and Lorentz invariants made of only first derivative of T.

A.1 Counting the number of invariants

Start with $N = 2n$ real tachyons in D spacetime dimensions, we have a general linear tachyon profile $T_i = a_{ij}X_j$ with ND parameters a_{ij}. Let us consider $N < D$ only. We can always go to a N-dimensional subspace in D dimensions so we can reduce a_{ij} to N^2 parameters ($i, j = 1, 2, \ldots, N$). Spacetime rotation $O(N)$ has $N(N-1)/2$ generators and $\mathrm{U}(n)$ has n^2 generators. We can further rotate (both spacetime and gauge) the linear profile to $K = N^2 - N(N-1)/2 - n^2$ independent parameters. For the brane-antibrane case, n=1 and N=2, so there are $K = 4 - 1 - 1 = 2$ parameters. For the U(2) case, we have $n = 2$ and $N = 4$, so there are $K = 16 - 6 - 4 = 6$ parameters. We may choose them to be $u_1 = a_{11}$, $u_2 = a_{22}$, $u_3 = a_{33}$, $u_4 = a_{44}$ as well as 2 cross terms u_5 and u_6. Therefore, based on the linear tachyon profile we expect that 6 invariants will be needed even though we will have only 4 eigenvalues in our $\mathcal{F}$s.

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The U(1) case is easy to solve; there are only 2 independent U(1) and Lorentz invariants:

$$\mathcal{X} \equiv 2\pi\alpha'^2 g^{\mu\nu}\partial_\mu T \partial_\nu \overline{T}\,, \qquad \mathcal{Y} \equiv \left(2\pi\alpha'^2\right)^2 \left(g^{\mu\nu}\partial_\mu T \partial_\nu T\right) \left(g^{\alpha\beta}\partial_\alpha \overline{T} \partial_\beta \overline{T}\right).$$

That is, other invariants that involve only first derivatives can be written in terms of these two.

In the case of U(2), there are 4 real tachyon fields, or, equivalently, a complex doublet. We will denote the two components of T by the complex T_1 and T_2. The 2 basic SU(2) invariant polynomials are the mesons and the baryons. For U(1) invariance, each baryon must be accompanied by an antibaryon. Such a product can be re-expressed as the antisymmetric sum of mesons :

$$\begin{aligned} M_{\mu\nu} &= \partial_\mu T^i \partial_\nu \overline{T}_i \\ B^{\mu\nu} &= \epsilon^{ij}\partial^\mu T_i \partial^\nu T_j \\ \tilde{B}_{\alpha\beta} &= \epsilon^{ij}\partial_\alpha \overline{T}_i \partial_\beta \overline{T}_j \\ B^{\mu\nu}\tilde{B}_{\alpha\beta} &= M_\alpha^{[\mu} M_\beta^{\nu]}\,, \end{aligned} \tag{A.1}$$

where the Latin letters are SU(2) indices and greek letters are Lorentz indices. Repeated indices are summed. Requiring U(1) invariance allows us to write every gauge invariant polynomial in terms of mesons only. We would like to find the simplest 6 invariants such that all other invariants that involve only first derivatives can be expressed with them. We shall still use the baryons in our basis to simplify the formula.

A.2 Invariant polynomials

As we have learned in the U(1) case, there are two different ways of contracting Lorentz indices: we can create a SU(2) scalar by contracting T with the corresponding $\overline{T}$ or we can form a SU(2) tensor by contracting T with another T (and doing the same for $\overline{T}$). An invariant in which all the Lorentz contractions are done in such a way that T is always contracting with a $\overline{T}$ will be a "scalar type" term (with a subscript s). If all the contractions are of the "tensor type" we will use a subscript t. When we reach terms with 6 derivatives or more, there are invariants with both types of contractions; they will be labeled with a subscript m for mixed. As we shall see, there are 2 mixed terms of 8 derivatives long and they will be differentiated with a prime. Finally, the number of derivatives would be indicated as another subscript. Since this number is always even, the actual number of derivative terms is twice the number in the subscript. For example, the two terms in the U(1) case are M_{s1} and M_{t2}. M_{m4} means a mixed term of 8 derivatives long and B_{t4} means a baryon term contracted as a tensor of 8 derivatives long.

To simplify the notation further, we drop the derivative in front of T since it is always there, and we will denote Lorentz contraction by putting the two fields that are contracted into parenthesis. SU(2) contraction would be understood to be in cyclic order (first with the last, second with the third,...) for the mesons. The following brackets show the SU(2) contraction structure.

$$M_{t4} = \overbrace{(T\,\underbrace{T)(\overline{T}}\,\underbrace{\overline{T})(T}\,\underbrace{T)(\overline{T}}\,\overline{T})}\,. \tag{A.2}$$

name	polynomials
M_{s1}	$(T\overline{T})$
M_{s2}	$(T\overline{T})(T\overline{T})$
M_{t2}	$(TT)(\overline{TT})$
B_{s2}	$\epsilon_{ij}\epsilon_{mn}(T_i\overline{T}_m)(T_j\overline{T}_n)$
M_{s3}	$(T\overline{T})(T\overline{T})(T\overline{T})$
M_{m3}	$(TT)(\overline{T}T)(\overline{TT})$
M_{s4}	$(T\overline{T})(T\overline{T})(T\overline{T})(T\overline{T})$
M_{t4}	$(TT)(\overline{TT})(TT)(\overline{TT})$
M_{m4}	$(TT)(\overline{T}T)(\overline{TT})(T\overline{T})$
$M^{'}_{m4}$	$(TT)(\overline{T}T)(\overline{T}T)(\overline{TT})$
B_{t4}	$\epsilon_{ij}\epsilon_{kl}\epsilon_{mn}\epsilon_{ab}(T_iT_k)(\overline{T}_m\overline{T}_a)(T_jT_l)(\overline{T}_n\overline{T}_b)$
B_{m4}	$\epsilon_{ij}\epsilon_{kl}\epsilon_{mn}\epsilon_{ab}(T_iT_k)(T_j\overline{T}_a)(\overline{T}_mT_l)(\overline{T}_n\overline{T}_b)$

Table 2: Basic invariants up to 8 derivatives.

For baryons we will keep the SU(2) clearly identified with their ϵ tensor. As examples :

$$\begin{aligned} M_{s1} &= \partial_\mu T^i \partial^\mu \overline{T}_i \to (T\overline{T}) = (1\overline{1}) + (2\overline{2}) \\ M_{t2} &= \partial_\mu T^i \partial_\nu \overline{T}_i \partial^\mu T^j \partial^\nu \overline{T}_j \to \\ (TT)(\overline{TT}) &= (11)(\overline{11}) + (12)(\overline{21}) + (21)(\overline{12}) + (22)(\overline{22})\,. \end{aligned} \tag{A.3}$$

Where $(1,2)$ refers to our 2 complex tachyons. Note that $(12)(\overline{21}) = (21)(\overline{12})$. It is clear from this notation that we have six basic objects $(11),(22),(1\overline{1}),(2\overline{2}),(12),(1\overline{2})$ and their complex conjugates. We like to show that there are 6 independent invariants.

Up to 8 derivatives there are at most 12 independent invariants We list them here: Some of these terms can actually be expressed as a function of the others. We give here a list of these relations. First, there is an iterative relation between M_{sn}, $M_{s(n-1)}$ and $M_{s(n-2)}$. From the first two invariants of this form we can get all the others.[5]

$$M_{sn} = M_{s(n-1)}M_{s1} - \frac{(M_{s1}^2 - M_{s2})}{2}M_{s(n-2)}\,. \tag{A.4}$$

Using this relation we learn that M_{s3} and M_{s4} are not independent. Furthermore, we already know that the baryons should be related to the mesons. Those relations are easy to find and they are:

$$\begin{aligned} B_{s2} &= M_{s1}^2 - M_{s2} \\ B_{t4} &= M_{t2}^2 - M_{t4} \\ B_{m4} &= M_{m4} - 2M^{'}_{m4} + M_{t2}M_{s2}\,. \end{aligned} \tag{A.5}$$

Finally the last non trivial relation is:

$$M^{'}_{m4} = M_{s1}M_{m3} - \frac{B_{s2}M_{t2}}{2}\,. \tag{A.6}$$

[5]The same relations exist for M_{tn} but it will not be needed since we do not go beyond M_{t4} which is only the second invariant that is totally contracted as a tensor.

This gives us a total of 6 relations among 12 invariants. One can show that higher derivative invariants can be expressed in terms of these 12, so we have a basis of 6 independent invariants, say: $M_{s1}, M_{s2}, M_{t2}, M_{m3}, M_{t4}, M_{m4}$. As we shall see, this basis is not the most suitable choice.

A.3 Choice of a basis

We will choose a basis which looks simple in the linear tachyon profile. A general linear tachyon profile can be expressed in the real field basis by $T_i = a_{ij} X_j$ where i is a gauge group indices (U(2) in our case and i runs from 1 to 4) and j is the spacetime index which runs over the dimensionality of the space. As pointed out earlier, there is no lost of generality to restrict ourselves to a 4 dimensional subspace. Therefore the a_{ij} will represent a 4×4 matrix and by a $\mathrm{SO}(4) \times \mathrm{U}(2)$ rotation we can eliminate 10 off-diagonal elements of this matrix, leaving 6 non-zero matrix elements in a_{ij}. Note that, generically, it will not have the form ;

$$T \neq \begin{pmatrix} u_1 & u_5 & 0 & 0 \\ 0 & u_2 & 0 & 0 \\ 0 & 0 & u_3 & u_6 \\ 0 & 0 & 0 & u_4 \end{pmatrix} X$$

since further U(1) rotations can render it completely diagonal with only 4 non-zero entries. In other words, it is impossible to get this linear profile by a $\mathrm{U}(2) \times \mathrm{SO}(4)$ transformation on a generic linear profile.

For the rest of the analysis, we consider the following linear profile. (We shall comment on other choices later.)

$$T = \begin{pmatrix} u_1 & 0 & u_5 & 0 \\ 0 & u_2 & 0 & u_6 \\ 0 & 0 & u_3 & 0 \\ 0 & 0 & 0 & u_4 \end{pmatrix} X \, ,$$

where T and X are 4 vectors. It is important to realize that for this basis all the 6 'pieces' $((11), (22), \dots)$ are real. We define

$$\begin{aligned} (1\bar{1}) &= u_+ \, , \qquad & (2\bar{2}) &= v_+ \\ (11) &= u_- \, , \qquad & (22) &= v_- \\ (1\bar{2}) &= w_+ \, , \qquad & (12) &= w_- \, . \end{aligned} \tag{A.7}$$

Now using that $(11) \to \partial_\mu (T_1 + iT_2) \partial^\mu (T_1 + iT_2)$ in the real field basis we can express the last expressions in terms of $u_1, u_2, \dots$:

$$\begin{aligned} u_\pm &= u_1^2 + u_5^2 \pm (u_2^2 + u_6^2) \\ v_\pm &= u_3^2 \pm u_4^2 \\ w_\pm &= u_5 u_3 \pm u_6 u_4 \, . \end{aligned}$$

Using a matrix notation ($T = UX$) and, following [3], the functions that would appear in the lagrangian is $TT^\dagger = (XU)^T UX$. After doing the path integral, we will get 4 $\mathfrak{F}$s with

name	# of derivatives	invariants	linear profile
K_1	2	$\frac{1}{2}M_{s1}$	$\frac{1}{2}(u_+ + v_+)$
K_2	4	$\frac{1}{4}(M_{s2} - B_{s2})$	$\frac{1}{4}((u_+ - v_+)^2 + 4w_+^2)$
K_3	4	$\frac{1}{4}(M_{t2} + \sqrt{B_{t4}})$	$\frac{1}{4}(u_- + v_-)^2$
K_4	4	$\frac{1}{4}(M_{t2} - \sqrt{B_{t4}})$	$\frac{1}{4}((u_- - v_-)^2 + 4w_-^2)$
K_5	8	$(M'_{m4} - M_{m4})$	$(w_+(u_- - v_-) - w_-(u_+ - v_+))^2$

Table 3: Invariants.

the variables being the eigenvalues of U^TU. The eigenvalues of the system are (U^TU)

$$
\begin{aligned}
c_1 &= A + \frac{1}{2}\sqrt{C} \\
c_2 &= A - \frac{1}{2}\sqrt{C} \\
c_3 &= B + \frac{1}{2}\sqrt{D} \\
c_4 &= B - \frac{1}{2}\sqrt{D}\,,
\end{aligned}
$$

where

$$
\begin{aligned}
A &= \frac{1}{2}(u_1^2 + u_3^2 + u_5^2) \\
B &= \frac{1}{2}(u_2^2 + u_4^2 + u_6^2) \\
C &= (u_1^2 - u_3^2)^2 + u_5^2(2u_1^2 + 2u_3^2 + u_5^2) \\
D &= (u_2^2 - u_4^2)^2 + u_6^2(2u_2^2 + 2u_4^2 + u_6^2)\,.
\end{aligned}
$$

We can solve those eigenvalues in terms of 5 different terms in the basis (only 4 different combinations of 5 terms).

From those basic invariant polynomials, we can solve for the c's. Defining:

$$
\mathcal{X}_\pm \equiv K_1 \pm \sqrt{K_3} \qquad \mathcal{Y}_\pm \equiv K_2 + K_4 \pm \sqrt{4K_2K_4 - K_5}
$$

from which we get

$$
\begin{aligned}
4\pi\alpha'^2 c_1 &= \mathcal{X}_+ + \sqrt{\mathcal{Y}_+} \\
4\pi\alpha'^2 c_2 &= \mathcal{X}_+ - \sqrt{\mathcal{Y}_+} \\
4\pi\alpha'^2 c_3 &= \mathcal{X}_- + \sqrt{\mathcal{Y}_-} \\
4\pi\alpha'^2 c_4 &= \mathcal{X}_- - \sqrt{\mathcal{Y}_-}\,.
\end{aligned} \tag{A.8}
$$

Another choice of the tachyon linear profile, for example, is a symmetric matrix with 6 entries. It gives the same answer, suggesting that our answer is independent of the choice of the basis. We have also considered a matrix which takes the form with a single eigenvalue, say c_1, while the remaining 3×3 matrix is in the Jordan canonical form with 5 u_i. In this case, we see rather easily that c_1 precisely reproduces the same function of the invariants given above.

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Now we can write down our action (from BSFT with linear tachyon profile and no gauge field) for the DD$\overline{\mathrm{D}}$ in a general covariant way. The action is proportional to:

$$\mathcal{F}(\mathcal{X}_+ + \sqrt{\mathcal{Y}_+})\,\mathcal{F}(\mathcal{X}_+ - \sqrt{\mathcal{Y}_+})\,\mathcal{F}(\mathcal{X}_- + \sqrt{\mathcal{Y}_-})\,\mathcal{F}(\mathcal{X}_- - \sqrt{\mathcal{Y}_-})\,. \tag{A.9}$$

It turns out that this expression does not have enough symmetry to cancel all the square roots. This could be a sign of non-local physics or simply a limitation of our effective action. It is easy to show that this action reduces to D$\overline{\mathrm{D}}$ action if we set T_2 to 0. In that case, the cross-terms and the v's go to zero (no more baryons contributions in the invariants) in the linear profile and there is no longer any difference between K_3 and K_4. In term of the invariants, we now have that $K_1^2 = K_2$ and $K_5 = 0$. It is easy to see that $\mathcal{Y}_\pm = (K_1 \pm \sqrt{K_3})^2$ and we get:

$$\begin{aligned}
&\mathcal{X}_+ + \sqrt{\mathcal{Y}_+} \to 2(K_1 + \sqrt{K_3})\\
&\mathcal{X}_+ - \sqrt{\mathcal{Y}_+} \to 0\\
&\mathcal{X}_- + \sqrt{\mathcal{Y}_-} \to 2(K_1 - \sqrt{K_3})\\
&\mathcal{X}_- - \sqrt{\mathcal{Y}_-} \to 0\,.
\end{aligned} \tag{A.10}$$

This is exactly the two invariants found in [5] and two of the $\mathcal{F}$'s just disappear ($\mathcal{F}(0) = 1$).

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